INTRODUCTION
In this article, we highlight a new and powerful method for
low-speed airfoil design and synthesis. Our motivation is
to offer more simplified, and yet more powerful, methods
to define and synthesize low-speed airfoil geometry and
to calculate pressure and viscous forces and moments
(this article limited to inviscid flow).

We’ll leverage the power of the cubic spline, together with
a new formulation, to first define an airfoil with just one
“spline knot” on each upper and lower surface. Then
adding a few more points, we can obtain a compact,
smooth, and accurate model of any airfoil. In calculating
surface pressures with the well-known vortex-panel
method, we’ll introduce new and fundamental methods
which “slay the dragon of discontinuity” when it becomes
necessary to compute the panel-self-induced normal and
tangential velocities. Finally the new methods, once
reasonably validated with test data, are applied toward
the design of an efficient, high-lift laminar airfoil.

1.0 AIRFOIL GEOMETRY DEFINITION
Traditionally, airfoil geometry has been defined by a
lengthy tabulation of coordinates, (z/c) versus (x/c).
Typically, 70 or so points are necessary to mitigate both
the differences in interpolation methods and the risk of
oscillations in computed velocity near the leading edge.

An efficient and ultimately more accurate alternative is to
characterize the geometry with a handful of well-placed
points, then interpolating between points with a spline.
Choices then become the type of spline and the type of
coordinates. Herein we apply a cubic spline together with
the Glauert Coordinate, the latter representing the polar
angle (ϕ) taken clockwise from the lower trailing edge.
This approach enjoys “automatic” anchor points at the
leading and trailing edges, enabling airfoil construction
with as little as one “spline knot” on each upper/lower
surface (Figure 1.0-1). For any number of knots, this
approach ensures continuous curvature throughout,
including the leading edge where such continuity is
particularly important. EQ [1.0-1] relates the horizontal
coordinate (X ≡ x/c) and Glauert coordinate, the latter
used for cubic-spline interpolation of the vertical
coordinate (Z ≡ z/c) as shown in Figure 1.0-2.

\[ X = x/c = (1/2) (1 + \cos \phi) \]  

[1.0-1]

2.0 CUBIC SPLINE REVIEW AND RENEW
The cubic spline numerical method joins a series of
“knots,” each having coordinates (X,Y), with a series of
third-order polynomials, while preserving continuity of
the zeroth, first, and second derivatives at each internal knot.
Although with this method the third derivative is in general
not continuous, indeed exhibiting a “zig-zag” shape, we
postulate that airfoil geometry characterized with a cubic
spline will be in effect smooth, not only to the eye, but
also to the air flowing over it.

3.0 PANEL-METHOD OVERVIEW
Our linear-vortex-panel method follows the general
approach of Katz and Plotkin (3), but with a different
approach to calculate panel-to-panel and self-induced
velocities, accommodation of panel curvature, and mixed
(normal versus tangential) boundary conditions.

Our objective is to solve for the distribution of pressure
coefficient (c_p) and velocity along the airfoil surface in
inviscid, low-speed flow. Although such modeling is
“inviscid,” viscosity is in fact essential for the development
of lift by giving rise to the boundary layer and by enforcing
the well-known Kutta Condition at the trailing edge.
The boundary layer is herein modeled as an infinitely-thin continuous vortex surface with local vortex density ($\gamma$), the latter positive when rotating clockwise for "left-to-right" flow. Airfoil geometry and vortex density are non-dimensionalized, respectively, to the chord (c) and flight velocity ($v_o$). The non-dimensional distance ($S = s/c$) is integrated beginning at the lower trailing edge, including the effects of local curvature, as unit normal vectors ($n$) are computed. Applying boundary conditions, with self-induced velocities where applicable, we solve for the non-dimensional vortex strength ($G = \gamma/v_o$) at ($n_v$) vortex cores.

### 3.1 SPLINE-BASED VECTOR INTEGRATION

In Figure 3.1-1, the incremental sub vortex ($\gamma_k \Delta s$) induces a velocity ($v_k = \gamma_k \Delta s / (2\pi r_k)$) at node (i), the latter having unit normal and tangential vectors ($n_i$) and ($t_i$), respectively. In general, we apply the boundary condition whereby the velocity normal to the surface at node (i) is zero, and where such normal velocity is the vector sum thereof induced by the free stream and the vortex surface. With the aid of a cubic spline, vectors, and new methods herein for self-induced velocities, we integrate over the vortex surface for the non-dimensional normal velocity ($\overline{v_i}$) at each node (i). To avoid numerical difficulties near the trailing edge, we apply the tangential counterpart of this boundary condition.

### 3.2 PANEL-SELF-INDUCED NORMAL VELOCITY

At the center of panel (a-b) in Figure 3.2-1, we now compute the panel-self-induced normal velocity. The simplest study treats the panel as flat, but it can be shown that a fixed radius of curvature will have no effect. First, we characterize the local non-dimensional vortex density ($G=\gamma/v_o$) as parabolic with non-dimensional distance ($S$) from the panel center. $G(S)$ is then distilled to three isolated components representing the average, first derivative, and second derivative. By inspection, the effects of the average cancel. Also, paying careful attention to sign, we discover that the effects of the 2nd-derivative also cancel, whereby we need concern ourselves only with the 1st derivative ($dG/dS$), given by $[(G_b-G_a)/(S_b-S_a)]$. 

![Figure 3.1-1 Induced Velocity Vector Integrand Constituents](image1.png)

![Figure 3.2-1 Self-induced Normal Velocity Study](image2.png)
3.3 SELF-INDUCED TANGENTIAL VELOCITY

When integrating induced normal velocity, we find that the strong mutual influence of upper and lower panels adjacent to the trailing edge leads to relatively large integrands together with mid-panel sign changes. These phenomena locally destabilize the solution for vortex strength. An alternative boundary condition, based on the tangential induced velocity, essentially resolves this problem. As before, however, we encounter the need to compute panel-self-induced velocity when integrating effects of the vortex surface.

Assuming the vortex panel is flat, a first look suggests that the panel-self-induced tangential velocity is either zero or perhaps undefined. However, taking into consideration the finite thickness of the boundary layer, while recognizing that the local velocity of interest resides just outside the boundary layer, we find that the self-induced tangential velocity is not zero. Figure 3.3-1 derives the self-induced tangential velocity increment (\(dvt\)) which, integrated in the limit as the b'layer thins, yields the self-induced tangential velocity of EQ 3.3-1.

\[
\Delta \tau_{ii} \equiv \frac{\Delta v_t}{v_o} = \frac{2}{\rho_o} \gamma_i \int_0^{\pi/2} \frac{\gamma_i ds}{2\pi r} \cos \theta \\
= \frac{2}{\rho_o} \gamma_i \int_0^{\pi/2} \frac{ds}{2\pi r} dr d\theta \\
= \frac{\gamma_i}{2\rho_o} = \frac{G_i}{2} \quad [3.3-1]
\]

A similar analysis to that carried out for the normal velocity reveals for tangential velocity that the effects of (\(G'=dG/dS\)) cancel, whereas the effects of both the average and 2\(^{nd}\)-derivative remain. However, we will assume herein that the effects of the 2\(^{nd}\)-derivative on panel self-induced tangential velocity near the trailing edge can reasonably be neglected. To end this section, we note that a reasonable solution can be obtained applying the tangential boundary condition throughout, but at some expense in "numerical stiffness."

3.4 METHOD VALIDATION

The method obtains a good match with test data as shown for three airfoils in Figures 3.4-1 through 3.4-3, the first three sub-figures representing the geometry spline, vortex density, and pressure coefficient of the FX66 airfoil. Recall that the magnitude |G| is the ratio of local-to-flight velocity. For example, we see that the maximum upper-surface velocity is 60% greater than flight velocity. Looking more closely, we observe the stagnation point (G=0) to reside on the lower surface at about 0.5% of chord aft of the leading edge. At the trailing edge, the airflow slows down to 85% of flight velocity. With further inspection, we notice upper-surface laminar-to-turbulent transition at 48% chord (62% lower surface). By plotting either (G) or (1-c_p) versus (X), the upper surface naturally resides on top, without the need for scale inversion.
4.0 METHOD APPLICATION

We now apply the methods herein to obtain the PCS-001 laminar airfoil of Figure 4.1. Defined by 13 spline knots, it has the popular “laminar rooftop” feature for high lift and low drag. Selected knots on the upper forward surface were manually adjusted to get the “plateau” represented by constant pressure and constant velocity. Not yet knowing its stall characteristics, further analysis and testing are needed to assess its suitability for safe flight.